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Magnetic field decay via Hall drift in the crusts of neutron stars

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Magnetic field decay in neutron stars

Neutron stars have the strongest magnetic fields in the universe, up to perhaps 10^{15} G for magnetars, around 10^{12} G for young radio and X-ray pulsars and 10^{8}–10^{10} G for much older millisecond pulsars. The clear correlation between age and field strength suggests the field decays with time, but exactly how this decay occurs remains an open question.

In the case of single, strongly-magnetised neutron stars, the timescale of simple resistive (Ohmic) dissipation of ~10^{10} years has been found to be far greater than the characteristic observed decay timescale of 10^{7}–10^{8} years (Goldreich and Reisenegger 1992). Another mechanism is required by which a neutron star's field can decay on a shorter timescale.

The Hall drift effect

Hall drift, namely the advection of magnetic flux by the current associated with it, is important in systems where the magnetic field is strong enough to make the cyclotron frequency comparable to or greater than the collision rate, and bulk flow velocities are not much larger than the relative velocity of different charge carriers, associated to the electric current.

Jones (1988) first proposed that this effect might play an important role in the evolution of neutron star magnetic fields. Under some circumstances, fields may be large enough for the above conditions to be satisfied - particularly in the solid crust where the electrons are the only charge carriers. Since the Hall effect conserves magnetic energy, it has been argued that it acts by redistributing energy amongst the different modes and creating steep magnetic energy gradients or cascades, on which resistive (Ohmic) dissipation can act much more quickly than it would on a smooth field.

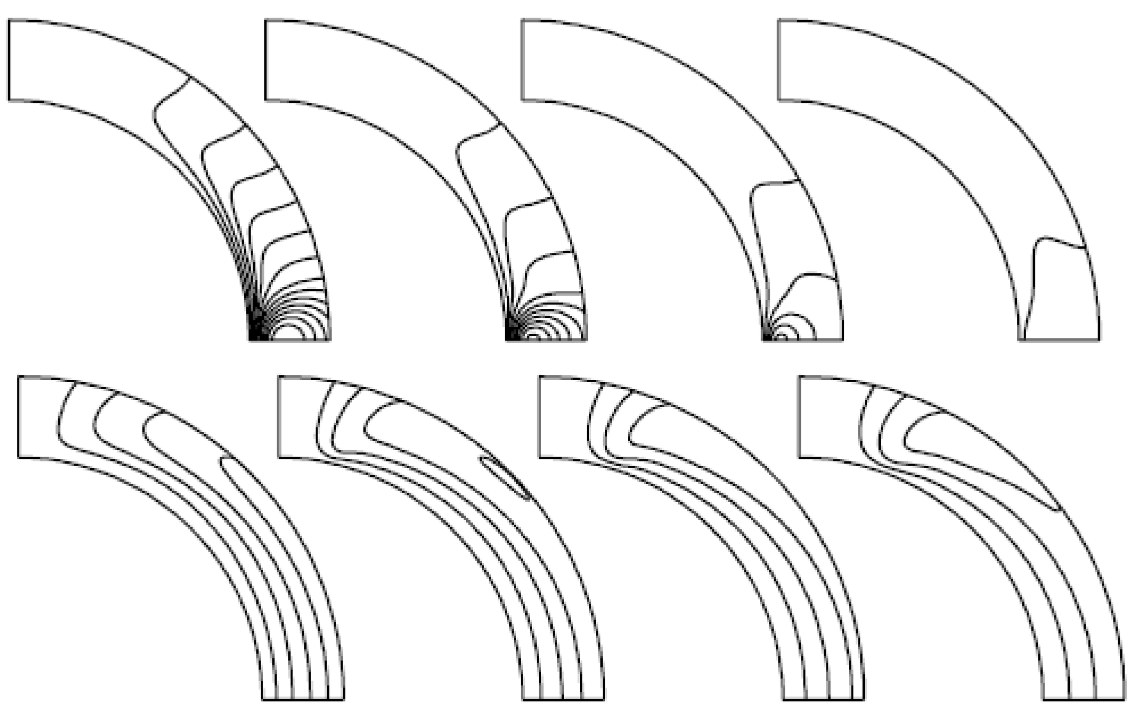

Figure 1. Contour plots of the poloidal field at t=0.25, 0.5, 0.75 and 1.0 from left to right. In the upper panel, results for the initial condition $-B_{tor} + 0.3\,B_{pol}$ and in the lower panel $B_{tor} + 0.05\,B_{pol}$. Note how the poloidal field evolves towards the equator with negatively signed toroidal field and towards the poles with positively signed toroidal field.

Previous work in this field

A number of authors have investigated whether Hall drift affects neutron star decay via numerical calculations (Shalybkhov & Urpin 1997; Urpin & Shalybkhov 1999; Hollerbach and Rűdiger 2002). All found that although Hall drift significantly affected the evolution of the field, in no case did it lead to decay on a timescale faster than Ohmic dissipation. However, these models were very idealized and did not include a number of additional features found in neutrons stars e.g. the very strong density gradient found in the crust. Vainshtein et al (2000) have shown that including this feature not only alters the Hall term in the governing equation, but does so in a way that may lead to rapid decay.

The Vainshtein et al model was limited in other ways, specifically to purely toroidal magnetic fields. Hollerbach and Rűdiger (2004) extended their previous model to include a combination of both poloidal and toroidal magnetic fields. They found that the Hall drift with stratification significantly affects the evolution of a neutron star's magnetic field, with the key features being the sign of the toroidal field in the two hemispheres (shown in Figure 1) and the ratio of poloidal to toroidal field.

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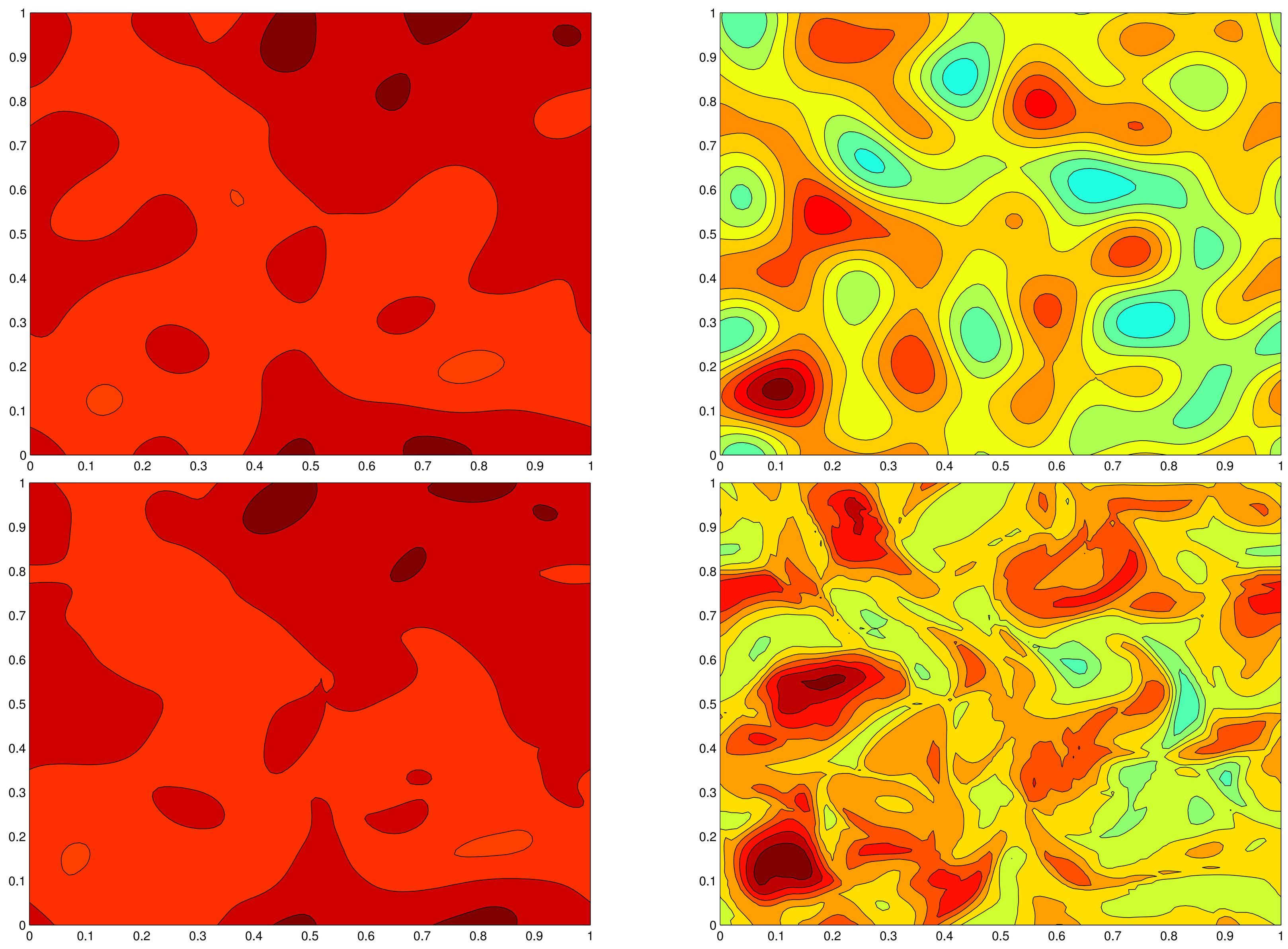

Figure 2. Contour plots of the poloidal (left column) and toroidal (right column) fields at t=0 (top row) and t=0.2 (bottom row). Note the rapid evolution from a smooth to complex field over a short timescale.

Computational method

Evolution of the magnetic field under the Hall effect is a non-linear process which has yet evaded full theoretical understanding. The dimensionless governing equation

$$\partial \mathbf{B}/\partial t \;=\; -\nabla \times [\,(\nabla \times \mathbf{B}) \times \mathbf{B}\,] \;+\; R_B^{-1}\nabla^2\mathbf{B} \qquad (1)$$

involves a ratio R_B between the non-linear Hall term (first on the right hand side) and the ohmic dissipation term (second). Our long term objective is to fully model this equation involving realistic conditions for the crust of a neutron star employing a 3D MHD spectral method. To this aim, we have currently extended the simulation limit of two decades of wavenumber to three and extended the limit of the R_B parameter from 400 to 3000. The results of this 2D modelling are shown in Figures 2 and 3.

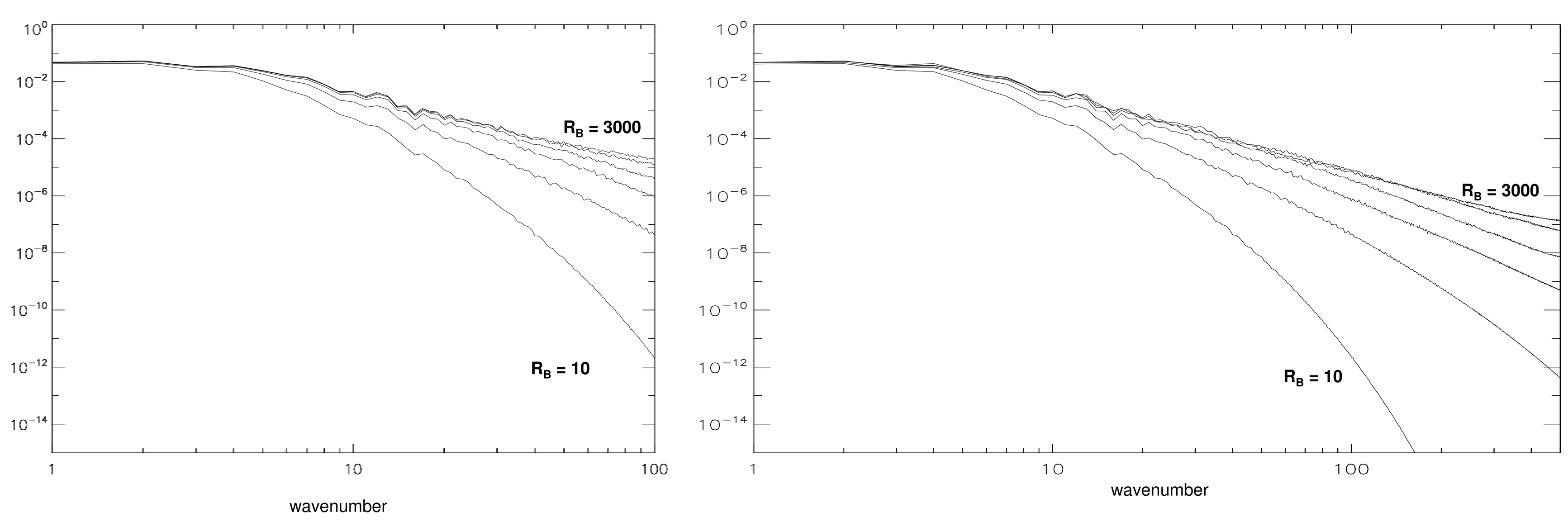

Figure 3. The spectrum of the field at t=0.2. Each panel shows results at R_B = 10,30,100,300,1000 and 3000. Note how the gradient decreases with increasing R_B: more energy is redistributed to higher wavenumbers where Ohmic dissipation can act more quickly at higher R_B. Left: a spatial resolution of 512^2 grid points. Right: a spatial resolution of 2048^2 grid points

Summary and future directions

We have found that the steep gradients found at lower wavenumbers and lower R_B extend similarly to high wavenumbers and realistic R_B (~1000). We have just completed development of a 3D version of this spectral method and are intending to investigate the same parameter space with realistic crust models of neutron stars. It is our aim to compare our theoretical results against observations of neutron stars, particularly if it possible to observe the exact alignment of the field, which we have found theoretically can be strongly dependent on the initial toroidal and poloidal fields.

References

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