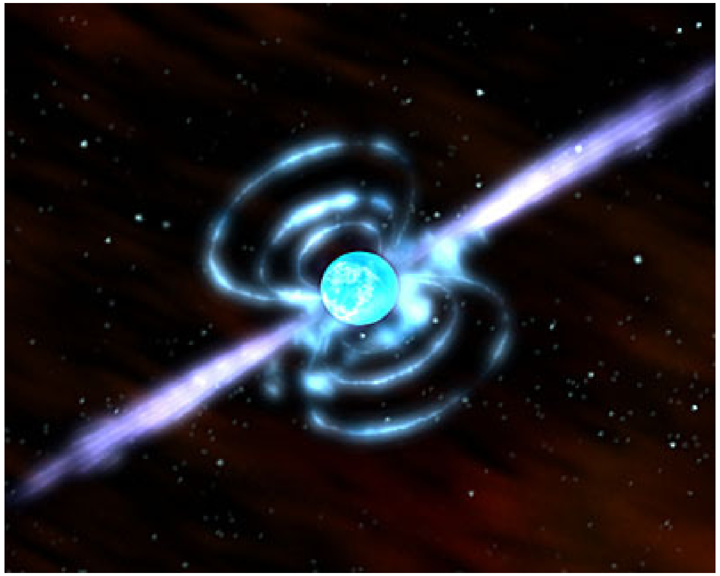
Forward and inverse cascades in 2D and 3D decaying electron MHD turbulence



UK MHD 2009

Thursday 4th June

Chris Wareing

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Overview

- What is electron MHD (EMHD)?
- Equations of EMHD.
- Our approach.
- 2D results.
- 3D results.
- Application to neutron stars.

Collaborators: Rainer Hollerbach, Steve Tobias.

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What is EMHD turbulence?

- Particular variety of plasma turbulence.
- Flow entirely due to electrons; ions form a static background.
- Also known as Hall MHD, or whistler turbulence.
- Applicable in many weakly collisional plasmas, e.g.
  - Sun’s corona;
  - Earth’s magnetosphere;
  - crusts of neutron stars.

Equations of EMHD turbulence

Governing equation, in application to the crusts of neutron stars:-

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [\mathbf{J} \times \mathbf{B}] + R_B^{-1} \nabla^2 \mathbf{B} \quad (1), \qquad \mathbf{J} = \nabla \times \mathbf{B}, \quad R_B = \sigma B_0 / nec$$

c.f. the vorticity equation governing ordinary, non-magnetic turbulence:-

$$\frac{\partial \mathbf{w}}{\partial t} = \nabla \times [\mathbf{u} \times \mathbf{w}] + \mathrm{Re}^{-1} \nabla^2 \mathbf{w} \quad (2), \qquad \mathbf{w} = \nabla \times \mathbf{u}$$

=> turbulent cascade, k^{-2} characteristic spectrum, dissipative cutoff at R_B^{-1}

Goldreich and Reisenegger (ApJ **395** 250 1992)

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Our approach

- Fundamental difference between (1) and (2)
  - Vorticity equation: dissipative term contains more derivatives than the nonlinear term.
  - EMHD equation: the two terms contain the *same* number of derivatives.

- The nonlinear term may dominate, even on arbitrarily short lengthscales.

- Previous work has employed hyperdiffusivity; clouds the issue at short lengthscales.

Numerical method

- Standard pseudospectral techniques to evaluate the nonlinear terms.
- De-aliasing according to the 2/3 rule.
- FFTW library to achieve massive parallelisation.
- Time integration using a second order Runge-Kutta method.
- Variety of 2D runs, N = 2048, R_B = 10, 30, 100, 300, 1000 & 3000.

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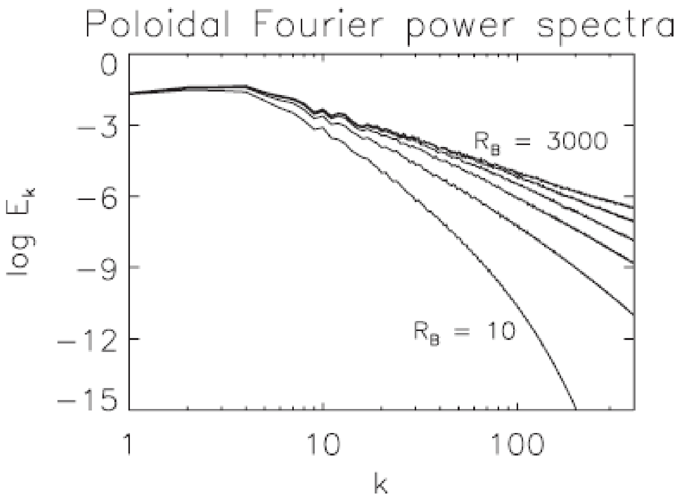
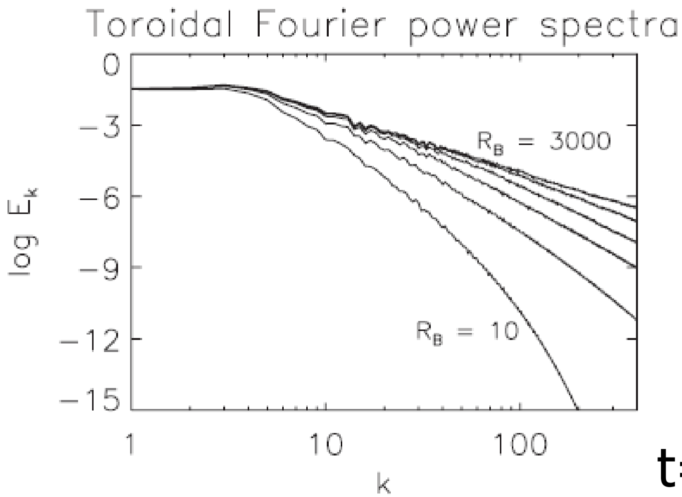
Results I – 2D free decay

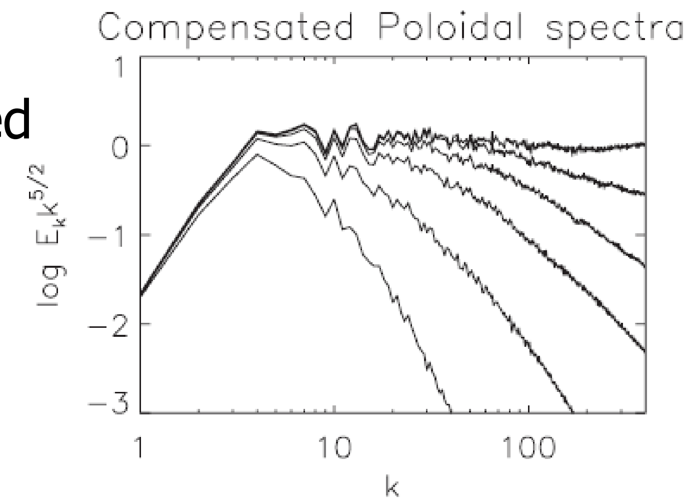
Fourier power spectra

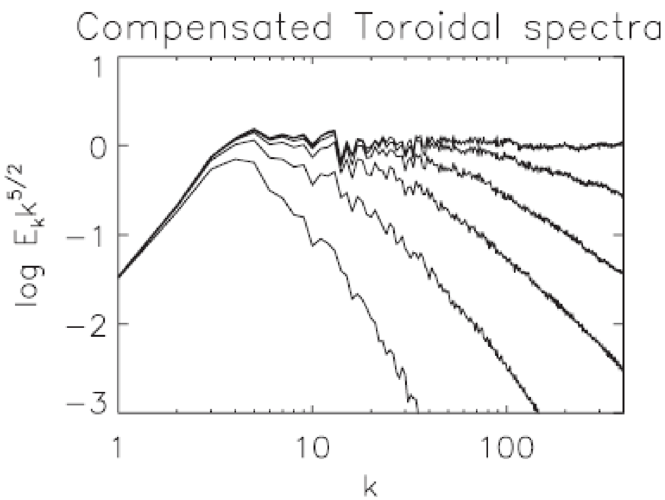




t=0.2

Compensated Fourier power spectra





- turbulent spectrum: $k^{-(5/2)}$, not $k^{-(7/3)}$!

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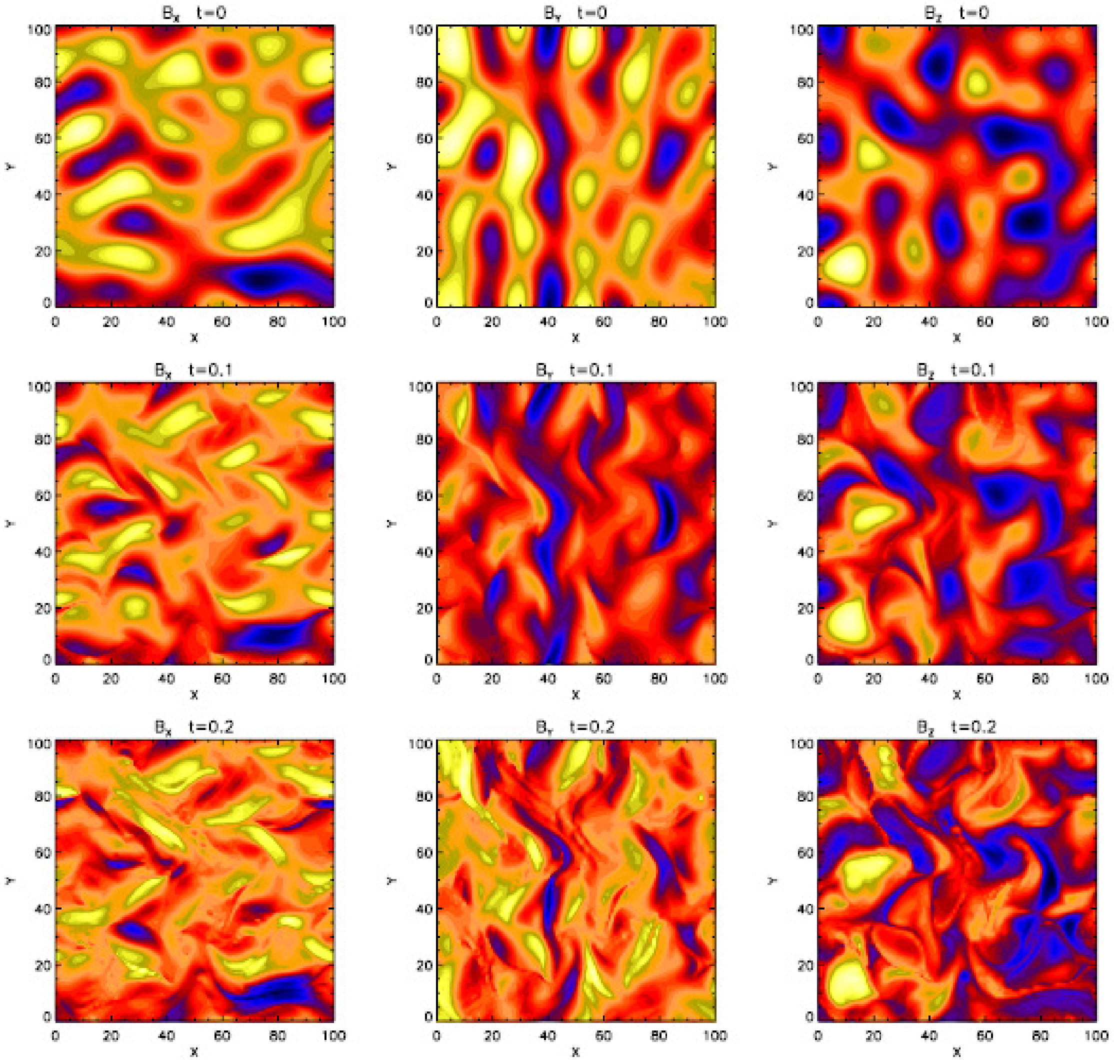
Results I – 2D free decay

Real space fields



t=0

t=0.1

t=0.2

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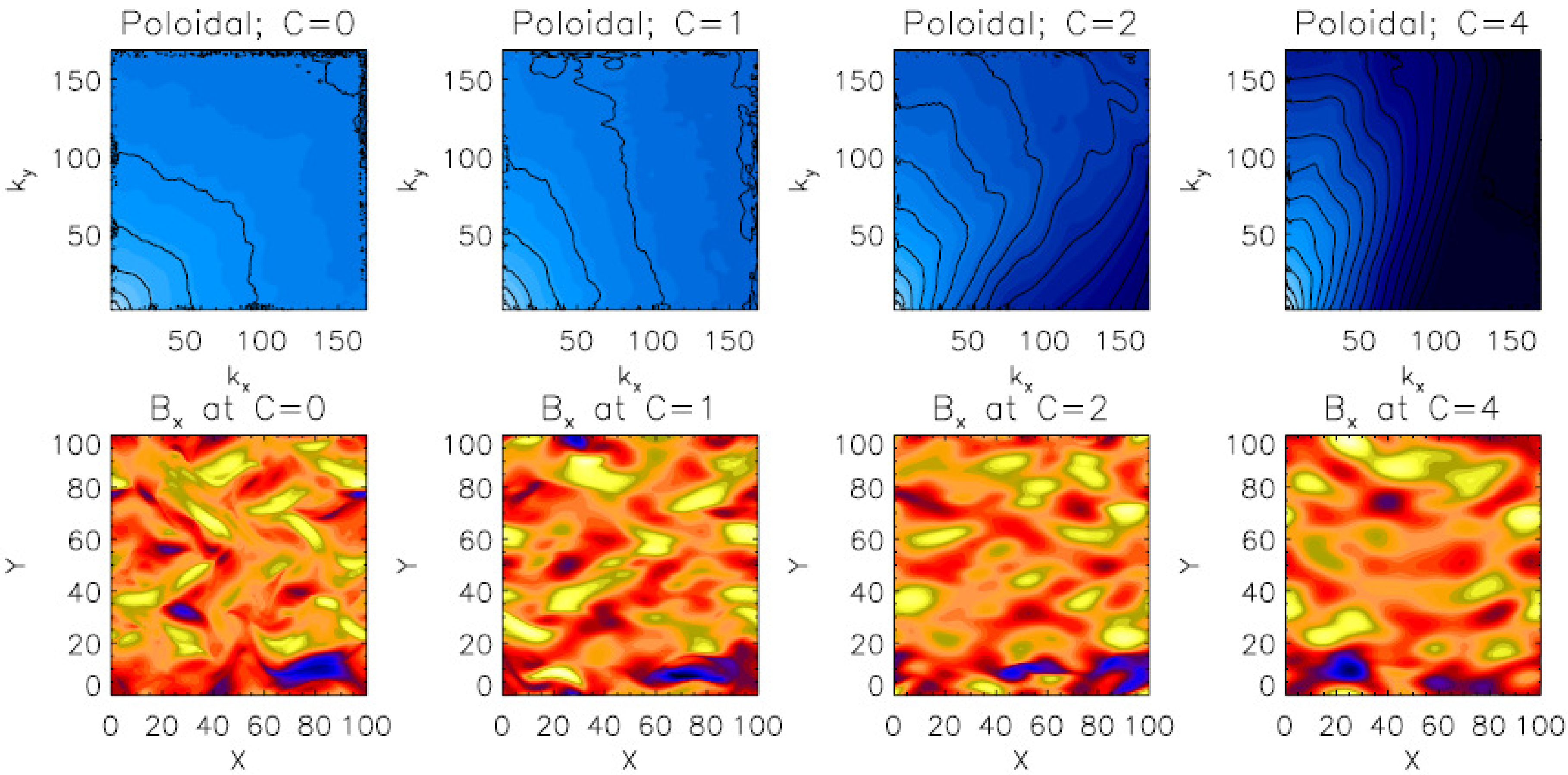
Results II – free decay in the presence of a background field

Uniform background field: $C\mathbf{e_x}$, t=0.2



Top row: 2D Fourier power spectra, bottom row: B_x real space field

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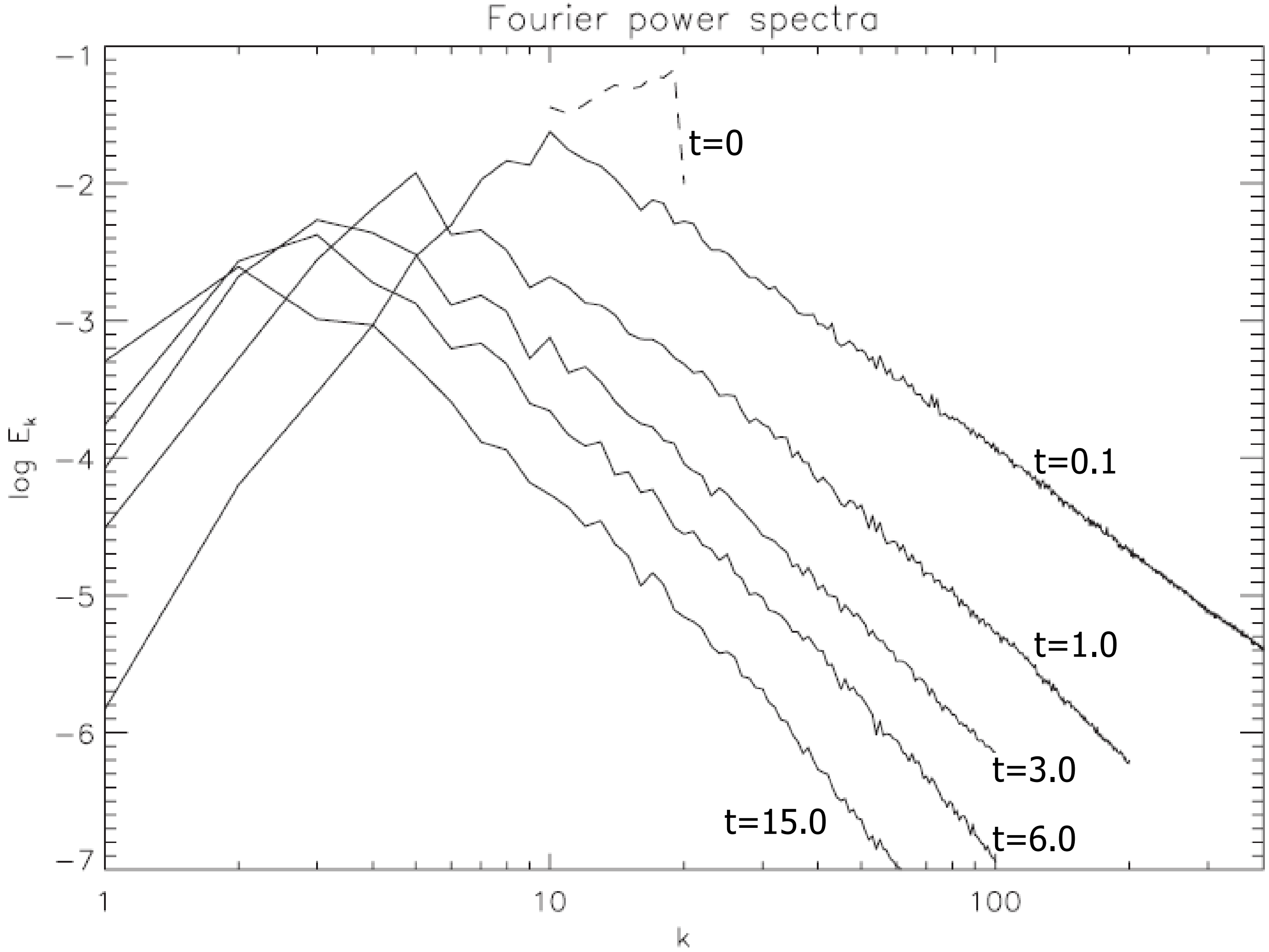
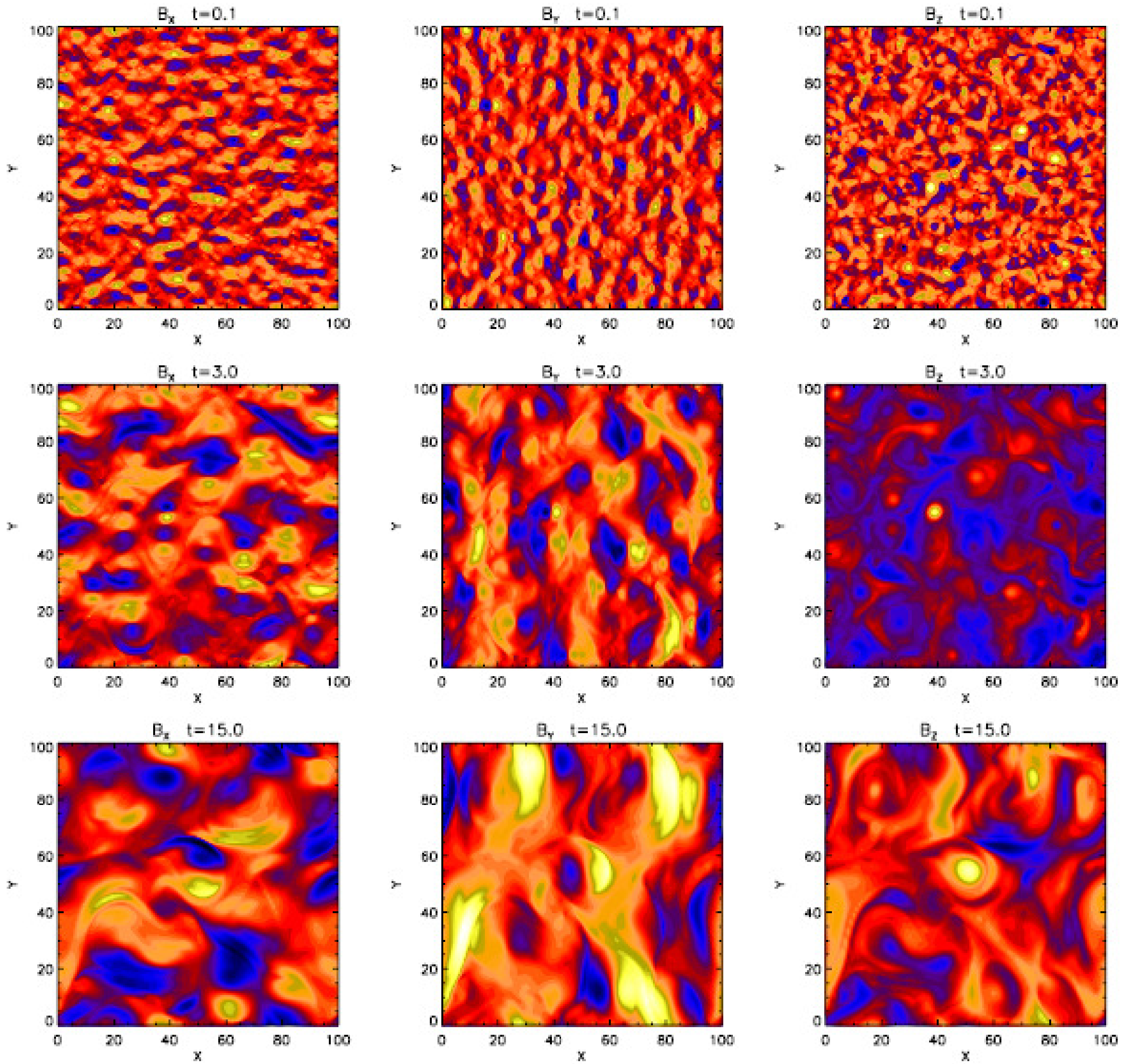
Results III – inverse cascade



Fourier power spectra

Results III – inverse cascade

Real space fields



t=0.1

t=3.0

t=15.0

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Conclusions I - 2D EMHD turbulence

- $k^{-5/2}$ scaling;
  c.f. previous estimates of -7/3
- No evidence for a dissipative cutoff;
  c.f. previous simulations found cutoff at $\sim R_B^{-1}$.

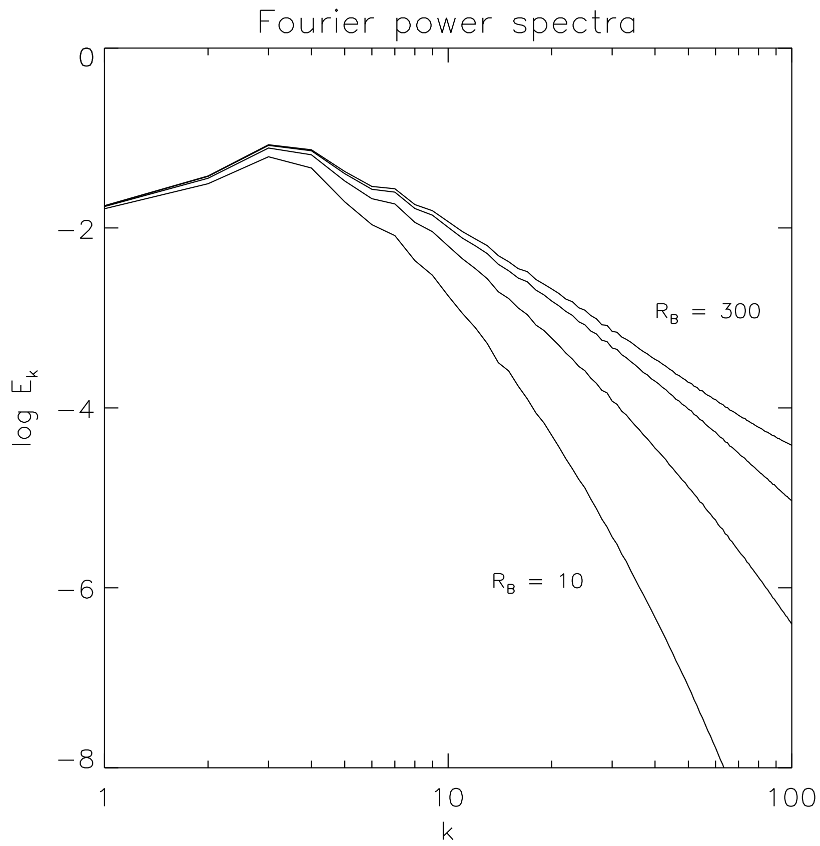
Hyperdiffusivity has clouded the issue of the equivalence of the terms in the governing eqn.

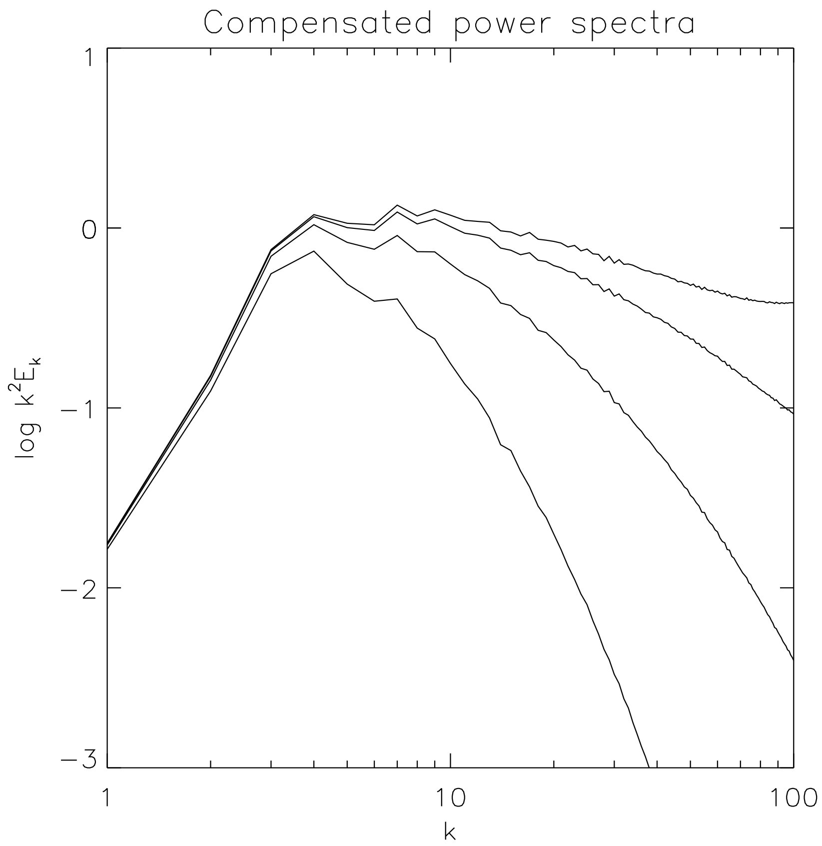
Coupling is non-local in Fourier space.

- Confirmed anisotropy in the presence of background field.
- First evidence for a strong inverse cascade of mean squared magnetic potential.

Results IV – 3D free decay

- Variety of 3D runs, N = 512, R_B = 10, 30, 100 & 300.





t=0.2

- turbulent spectrum: k^{-2}, not $k^{-(7/3)}$!

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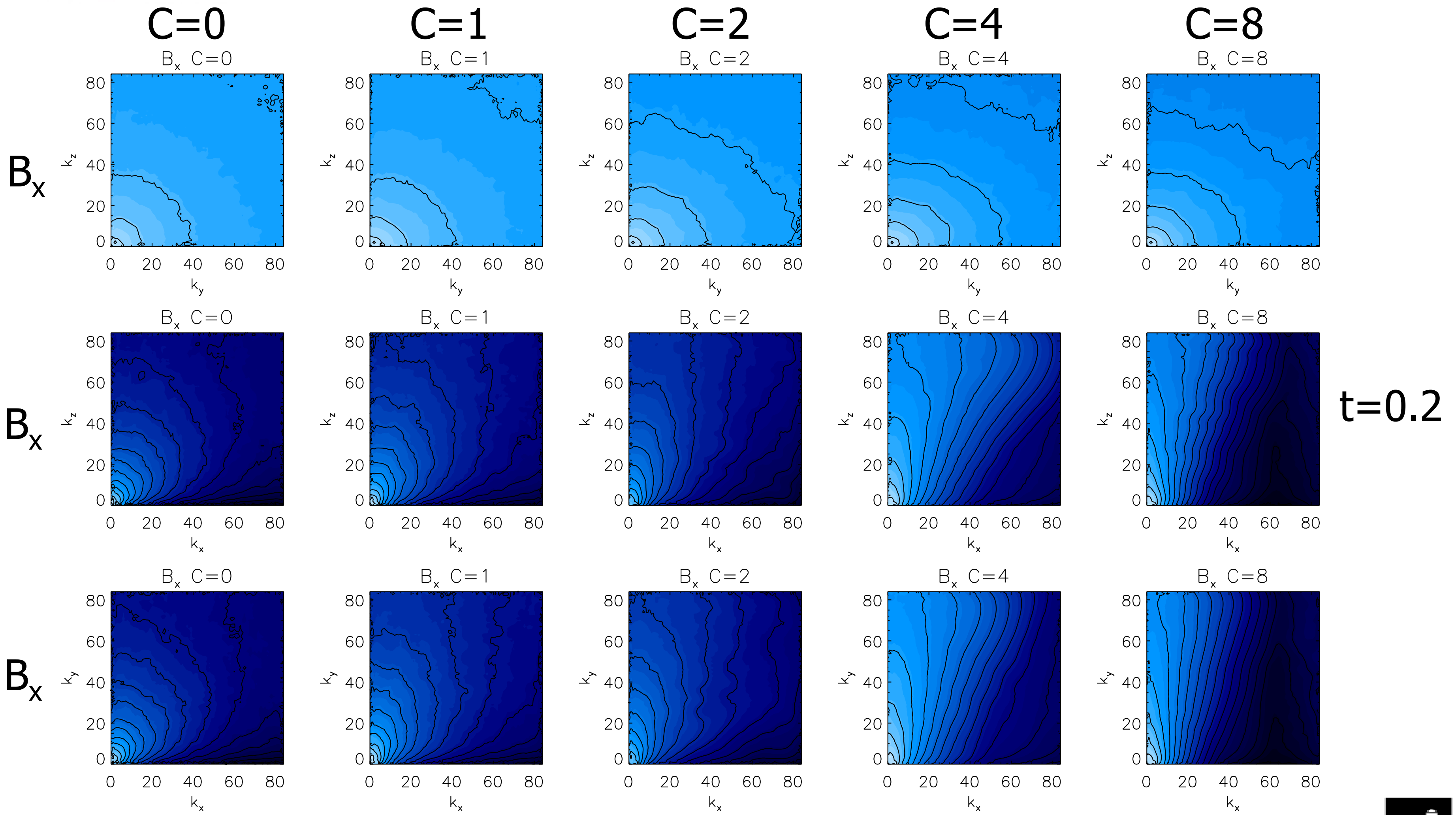
Results V – 3D free decay in the presence of a background field



Fourier power spectra, uniform background field: $C\mathbf{e}_x$

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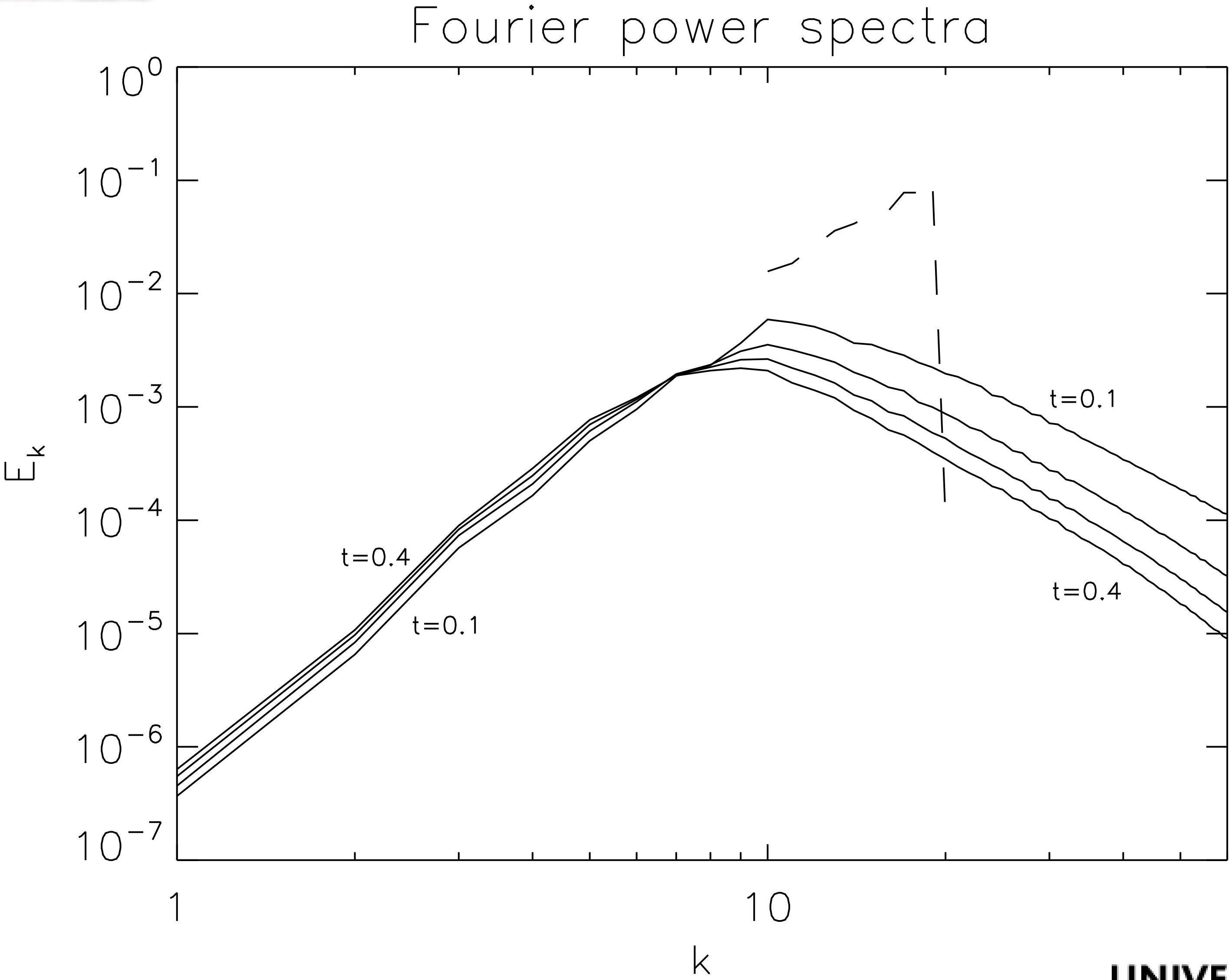
Results VI – 3D inverse cascade



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Conclusions II - 3D EMHD turbulence

- k^{-2} scaling; different to 2D

  c.f. previous numerical estimates of -7/3

  In agreement with Goldreich & Reisenegger (1992)

  Again, no evidence for a dissipative cutoff.

- Hyperdiffusivity has clouded the issue of the equivalence of the terms in the governing eqn.

  Coupling is non-local in Fourier space.

- Anisotropic in the presence of a background field.

- Very weak inverse cascade; not unexpected since mean squared magnetic potential is **not** conserved in 3D.

Application to neutron stars

- In the crusts of isolated neutron stars, the electrons are thought to move through a frozen lattice of atoms. Typical $R_B \sim 10^3$.

  - Hall decay rearranges the magnetic field energy.
  - Ohmic decay acts faster on the smaller lengthscales, thereby accelerating magnetic field decay.

- Since the governing equation is scale invariant, a very small box will see the large-scale field as a background field
  => expect the smallest scales in the system to be anisotropic.

- Inverse cascades may be the way in which a neutron star obtains a large scale ordered field from a primarily small scale disordered one.

Thank you for listening.

Any questions?

cjw@maths.leeds.ac.uk

2D results – Physics of Plasmas 16 042307 (2009)
3D results – Journal of Plasma Physics, *accepted*.
Application to neutron stars – *in preparation*.

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